# 6

# Nonlinear Localization of BECs in Optical Lattices

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## 6.1 Introduction

In this chapter we review the effects of spatial localization of a Bose–Einstein condensate (BEC) that arise due to the combination of the intrinsic nonlinearity of a condensate due to *repulsive* atomic interactions and Bragg scattering of a matter-wave on a periodic potential of an optical lattice (OL). It goes without saying that we will not discuss the trivial case of very deep periodic potentials which leads to trapping of the atoms within single sites. Instead, we will address the situation where the *single* particle tunneling rate is still much faster than the observation time. Under such conditions, a nontrivial localization becomes possible due to the fact that, at the edges of a Brillouin zone (BZ) of the lattice, the condensate experiences anomalous diffraction (dispersion), the magnitude of which can be controlled by tuning the depth of the OL potential. Keeping a wave packet from spreading can therefore be either achieved by actively controlling the dispersion or by utilizing the interaction between atoms. The first approach is known as dispersion/diffraction management and the second one leads to nonlinearly localized states.

The anomalous diffraction (dispersion) gives rise to the effects that are normally absent in a repulsive condensate. One of this effects is the modulational (dynamical) instability of the extended nonlinear Bloch waves that underpins the condensate localization, as well as leading to enhanced growth of the thermal fraction. The other effect is the spatial localization of a repulsive condensates inside the linear band gaps. The localized states take the form of gap solitons, for low atom numbers, and self-trapped states, for high atom numbers. Here we describe both the experimental observations and theoretical studies of the dispersion management, as well as formation and properties of nonlinear localized states of the BECs in 1D OLs. Although the recent progress in the studies of nonlinear localization is driven by the experiments with 1D OLs, we also summarize theoretical predictions of novel localized states with a nontrivial phase that may exist in 2D and 3D lattices.

## 6.2 Experimental Work Horse: Optical Potentials

The details concerning the experimental realization and detection of ultracold gases can be found in literature [1] and will not be addressed here. In this Section we will briefly recapitulate how OLs, i.e. periodic potentials, are realized and what parameters are under experimental control.

The success of atomic gases as an experimental system for the investigation of many different aspects of physics follows from the fact that atoms resonantly interact with light [2]. For example, the spontaneous emission is the prerequisite for the implementation of laser cooling since it allows us to dissipate kinetic energy of the atoms. In contrast to this incoherent process, the coherent redistribution of photons between different light modes via stimulated emission leads to the conservative potentials known as light shift or dipole potentials for atoms. In a classical picture these potentials are a consequence of the interaction of the dipole induced by the light with the light field itself. As the main result one finds that the ensuing potential is proportional to the *light intensity* and inversely proportional to the frequency difference between the laser light and the atomic transition. Since the decoherence, or incoherent spontaneous emission process, scales inversely with the *square* of the frequency difference, it is always possible to realize a situation such that the incoherent processes are negligible.

The optical dipole potentials are thus as variable as the light intensity $I(\boldsymbol{r})$ distributions. Using $L$ light beams impinging on the atoms from different directions $\boldsymbol{k}_j$, and with different amplitudes $\boldsymbol{E}_j$, potentials $V(\boldsymbol{r})$ of the form:

$$V(\boldsymbol{r}) \propto I(\boldsymbol{r}) = \frac{c\epsilon_0}{2} \sum_{j,l=1}^{L} \boldsymbol{E}_j \boldsymbol{E}_l^* \mathrm{e}^{\mathrm{i}(\boldsymbol{k}_j - \boldsymbol{k}_l)\cdot\boldsymbol{r}}, \quad \boldsymbol{E}(\boldsymbol{r}) = \sum_{j=1}^{L} \boldsymbol{E}_j \mathrm{e}^{\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{r}}, \qquad (6.1)$$

can be realized. Some examples of the resulting light distributions, i.e. potentials for the atoms, are shown in Fig. 6.1. It is important to understand that each pattern results from many beam interferences (see (6.1)), and thus it is obvious that the pattern depends on the relative phase between the beams. In the case of two and three beam interference this only leads to a shift of the pattern while the 'light crystal' symmetry is invariant under the phase change. For more beams, the phase change, as indicated on the right-hand side in Fig. 6.1, leads to different pattern shapes. In the real experiment, controlling the phase implies a mechanical stability of the light path lengths much better than a wavelength. However, this sensitivity is easily suppressed by using the fact that the atoms have inertia and hence can only react to potential changes at a finite time. The desired pattern is usually built up with standing light waves (two beam interference), i.e., periodic potentials, but the frequencies of the standing light waves are chosen differently. Therefore, the cross interference terms are not stationary but lead to moving patterns. As we will see below, the resulting velocity is directly proportional to the frequency difference. Using MHz frequency differences, straightforwardly realized by acousto